Local risk minimization for divident streams, BSDE approach

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1. Problem
2. Hedging
3. Local risk minimization
4. Local risk minimization BSDE

Statement of the problem

Let us consider market with following investment opportunities:

1. d- Stocks with price process denoted by S.
2. Bank account denoted by B.

Suppose that investor just sold contract for delivering cashflows according to the process D Investor is interested in hedging of D

Example (Call option)

$$D_t = \mathbb{1}_{\{t \geq T\}}(S_T - K)^+$$

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Example (Vulnerable Call option)

Let τ be a default time of the corporate. Suppose that $(S_T - K)^+$ is paid at maturity provided that $\tau > T$.

$$D_t = \mathbb{1}_{\{t \geq T\}} (S_T - K)^+ \mathbb{1}_{\{\tau > T\}}$$

Example (Vulnerable Call option with recovery)

1. $(S_T - K)^+$ is paid at the maturity of the option contract provided that the stock did not defaulted by T i.e. $\tau > T$.
2. fixed fraction $\delta \in [0, 1]$ of intrinsic value of the option is paid at the default time τ

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Suppose that there is additional source of uncertainty i.e. process C taking values in a finite set $\mathcal{K} = \{1, \ldots, K\}$ which can be interpreted as credit rating of some corporate or as state of economy. For each $i \in \mathcal{K}$ we define the processes

$$H_t^i := 1_{\{i\}}(C_t)$$

$$H_t^{i,j} := \sum_{0<u\leq t} H_{u-}^i H_u^j$$

$$D_t = \mathbb{1}_{\{t\geq T\}} h(S_T, C_T) + \int_0^t g(S_u, C_u) du + \sum_{i,j\in\mathcal{K}, i\neq j} \delta^{i,j}(S_{u-}) dH_u^{i,j}$$

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Hedge dividend stream ?

1. Perfect replication of dividend stream: self-financing strategy $\varphi = (\phi, \eta)$ such that

$$D_T - D_t = V_t(\varphi) + \int_t^T \phi_u dS_u + \int_t^T \eta_u dB_u$$

or in discounted terms

$$D_T^* - D_t^* := \int_t^T B_u^{-1} dD_u = V_t(\varphi) + \int_t^T \phi_u dS_u^*$$

2. Mean-variance hedging - minimize distance between the payoff and the gains from strategy (see Schweizer).
3. Risk/local-risk minimization - minimize at the risk measured by conditional second moment of remaining cost see Föllmer and Sonderman (1986) or Schweizer (2008). Replication is perfect but the strategy is not self-financing !

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Definition

Fix a payment stream D. The (cumulative) discounted cost process of an L^2 strategy $\varphi = (\phi, \eta)$ is

$$C_t^D(\varphi) := D_t^* + V_t^*(\varphi) - \int_0^t \phi_s dS_s^*, \quad 0 \le t \le T.$$

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L^2 strategy φ is called self-financing for D if $C^D(\varphi)$ is constant, and mean self-financing if $C^D(\varphi)$ is martingale (square integrable)

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The risk process of φ is

$$R_t^D := \mathbb{E}\left(\left(C_T^D(\varphi) - C_t^D(\varphi)\right)^2 |\mathcal{F}_t\right), \quad 0 \le t \le T.$$

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Assume that discounted prices are given by a $\mathbb{R}^d$ valued cadlag semimartingale.

Definition

We say that S^* satisfies structure condition (SC) if

1. S^* is special with canonical decomposition $S^* = S_0^* + M + A$ where $M \in \mathcal{M}_{0,loc}^2$
2. There exists $\mathbb{R}^d$ valued stochastic process a s.t.

$$A_t = \int_0^t d\langle M \rangle_s a_s$$

where λ is predictable and in $L_{loc}^2(M)$ i.e.

$$\int_0^T a_s^\top d\langle M \rangle_s a_s < \infty \qquad \mathbf{P} - a.s.$$

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Definition

The space Θ_S consist of all $\mathbf{R}^d$ valued predictable stochastic process ϕ s.t. the stochastic stochastic integral $\int \phi dS^*$ is well defined and in space $\mathcal{S}^2(P)$ of semimartingales. This means that

$$\mathbb{E}\left(\int_0^T \phi_s^\top d\langle M\rangle_s \phi_s + \left(\int_0^T |\phi_s dA_s|\right)^2\right) < \infty$$

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An L^2 strategy is a pair $\varphi = (\phi, \eta)$ where $\phi \in \Theta_S$ and η is real-valued adapted process s.t. $V^*(\varphi) := \phi^\top S^* + \eta$ is right-continuous square integrable i.e. $V_t^*(\varphi) \in L^2(P)$. If $V_T(\varphi) = 0$ $\mathbf{P} - a.s.$ then we say that φ is 0-achieving.

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An L^2 strategy is a pair $\varphi = (\phi, \eta)$ where $\phi \in \Theta_S$ and η is real-valued adapted process s.t. $V^*(\varphi) := \phi^\top S^* + \eta$ is right-continuous square integrable i.e. $V_t^*(\varphi) \in L^2(P)$. If $V_T(\varphi) = 0$ $\mathbf{P} - a.s.$ then we say that φ is 0-achieving.

Definition

The space Θ_S consist of all $\mathbf{R}^d$ valued predictable stochastic process ϕ s.t. the stochastic stochastic integral $\int \phi dS^*$ is well defined and in space $\mathcal{S}^2(P)$ of semimartingales. This means that

$$\mathbb{E}\left(\int_0^T \phi_s^\top d\langle M\rangle_s \phi_s + \left(\int_0^T |\phi_s dA_s|\right)^2\right) < \infty$$

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Characterization of locally risk minimization strategies for dividend

Theorem (Schweizer (2009))

Suppose the $\mathbf{R}^d$–valued semimartingale S^* satisfies structure condition (SC) and let D be a payment stream. If the mean-variance trade-off process

$$K_t = \int_0^t \alpha_u^\top d\langle M\rangle_u \alpha_u$$

is continuous, the following conditions are equivalent for an L^2 strategy φ:

1. *φ is locally risk-minimizing for D.*
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Föllmer-Schweizer decomposition of random variable Y

Definition

An $\mathcal{F}_T$ measurable random variable $Y \in L^2$ admits a Föllmer-Schweizer decomposition if it can be written as

$$Y = Y^{(0)} + \int_0^T \phi_s^Y dS_s^* + L_T^Y \qquad \mathbf{P} - a.s.$$

where $Y^{(0)} \in L^2$ is $\mathcal{F}_0$–measurable, $\phi^Y \in \Theta_S$, and the process L^Y is a (right-continuous) square integrable martingale null at zero and strongly orthogonal to M.

Remark

If S^* is a square integrable martingale then Föllmer-Schweizer decomposition is a Galtchouk-Kunita-Watanabe decomposition.

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Föllmer-Schweizer decomposition and locally risk minimization strategies

Theorem (Schweizer (2009))

Suppose the R^d–valued semimartingale S^* satisfies (SC) and the process K is continuous. Then a payment stream D admits a locally risk-minimizing L^2 strategy φ if and only if D_T^* admits a Föllmer-Schweizer decomposition. In that case $\varphi = (\phi, \eta)$ is given by

$$\phi = \phi^{D_T^*}, \qquad \eta = V^* - (\phi^{D_T^*})^\top S^*$$

with

$$V_t^* := D_T^{*(0)} + \int_{]0,t]} \phi_s^{D_T^*} dS_s^* + L_t^{D_T^*} - D_t^*, \qquad 0 \le t \le T,$$

and then

$$C_t^D(\varphi) = D_T^{*(0)} + L_t^{D_T^*}, \qquad 0 \le t \le T.$$

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Market model

Money market account satisfies

$$dB_t = B_t r_t dt, \quad B_0 = 1.$$

where r is bounded progressively measurable stochastic process. The dynamics of discounted price process $S_t^* := S_t/B_t$ is given by

$$dS_t^* = \mu_t dt + \sigma_t dW_t + \int_{\mathbf{R}^n} F_t(x)\widetilde{\Pi}(dx, dt) + \sum_{i,j\in\mathcal{K}, j\neq i} \rho_t^{i,j} dM_t^{i,j}, \quad S_0^* = s.$$

W is an n dimensional Wiener process,
by $\widetilde{\Pi}$ we denote

$$\widetilde{\Pi}(dx, du) := \Pi(dx, du) - \nu_u(dx)du,$$

where $\Pi(dx, dt)$ is assumed to be an integer valued random measure on $\mathcal{B}(\mathbf{R}^n) \otimes \mathcal{B}(\mathbf{R}_+)$ with compensator $\nu_u(dx)du$.

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Processes $M^{i,j}$ are driven by an additional source of uncertainty i.e. by a càdlàg process C taking values in a finite set $\mathcal{K} = \{1, \dots, K\}$ which can be interpreted as credit rating of some corporate or as state of economy. For each $i \in \mathcal{K}$ we define the processes

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We say that $(W, \widetilde{\Pi}, M)$ has a weak property of predictable representation with respect to $(\mathbb{F}, \mathbb{P})$ if every square integrable $(\mathbb{F}, \mathbb{P})$-martingale N has the representation

$$N_t = N_0 + \int_0^t \phi_u dW_u + \int_0^t \int_{\mathbf{R}^n} \psi_u(x) \widetilde{\Pi}(du, dx) + \int_0^t \sum_{i,j \in \mathcal{K}: j \neq i} \xi_u^{i,j} dM_u^{i,j},$$

where ϕ_u, $\psi_u(x)$, $\xi_u^{i,j}$ are predictable processes such that

$$\mathbb{E}\left(\int_0^T |\phi_u|^2 du \right) < \infty, \quad \mathbb{E}\left(\int_0^T \int_{\mathbf{R}^n} |\psi_u(x)|^2 \nu_u(dx) du \right) < \infty,$$

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$$N_t = N_0 + \int_0^t \phi_u dW_u + \int_0^t \int_{\mathbf{R}^n} \psi_u(x) \widetilde{\Pi}(du, dx) + \int_0^t \sum_{i,j \in \mathcal{K}: j \neq i} \xi_u^{i,j} dM_u^{i,j},$$

where ϕ_u, $\psi_u(x)$, $\xi_u^{i,j}$ are predictable processes such that

$$\mathbb{E}\left(\int_0^T |\phi_u|^2 du\right) < \infty, \quad \mathbb{E}\left(\int_0^T \int_{\mathbf{R}^n} |\psi_u(x)|^2 \nu_u(dx) du\right) < \infty,$$

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Assumption INT. The processes μ^i are predictable with values in the space of vectors of dimension d, σ^i are predictable with values in the space of matrices of dimension $d \times n$ and $\rho^{i,j}$ are predictable processes with values in the space of vectors of dimension d satisfying

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For convenience we denote by G the matrix valued stochastic process

$$G_t := \sigma_t(\sigma_t)^\top + \int_{\mathbf{R}^d} F_t(x)(F_t(x))^\top \nu_t(dx) + \sum_{i,j \in \mathcal{K}: j \neq i} \rho_t^{i,j} \left(\rho_t^{i,j}\right)^\top H_{t-}^i \lambda_t^{i,j}.$$

(SC) condition in this case means that $\mu_t \in Im(G_t)$ so that we can write

$$A_t = \int_0^t d\langle M \rangle_u a_u = \int_0^t G_u a_u du,$$

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Suppose that we have contract with dividends described by following integral equation

$$D_t = D_0 + \mathbb{1}_{t \geq T} h^D_T + \int_0^t g^D_u du + \int_0^t (\delta^D_u)^\top dW_u + \int_0^t \int_{\mathbb{R}^d} J^D_u(x) \widetilde{\Pi}(du, dx) + \int_0^t \sum_{i,j:j \neq i} \gamma^{D,i,j}_u dM^{i,j}_u$$

and let us define

$$A_t(\delta, J, \gamma) := \left(\sigma_t \delta + \int_{\mathbf{R}^n} F_t(x) J(x) \nu_t(dx) + \sum_{j,i \in \mathcal{K}, j \neq i} \rho^{i,j}_t \gamma^{i,j} \lambda^{i,j}_t \right)$$

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Theorem

Suppose that $(V, \delta^V, J^V, \gamma^V)$ *is a solution to following BSDE*

$$V_t = h_T + \int_t^T g_u^D - a_u^\top A_u(\delta_u^V + \delta_u^D, J_u^V + J_u^D, \gamma_u^V + \gamma_u^D) - V_u r_u du$$
$$- \int_t^T (\delta_u^V)^\top dW_u - \int_t^T \int_{\mathbf{R}^n} J_u^V(x)\widetilde{\Pi}(dx, du) - \int_t^T \sum_{j,i\in\mathcal{K}, j\neq i} \gamma_u^{V,i,j} dM_u^{i,j}$$

then (V_0, ϕ, L) *is a Fölmer-Schweizer decomposition of random variable* D_T^*, *where*
1) ϕ *solves following system of linear equations*

$$G_t\phi_t = A_u(\delta_u^V + \delta_u^D, J_u^V + J_u^D, \gamma_u^V + \gamma_u^D)$$

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$$L_t = \frac{V_t}{B_t} - V_0 - \int_0^t \phi_u dS_u^* - D_t^*$$

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Remark

First component of solution to BSDE is a value process of locally risk-minimizing strategy.

Remark

The martingale L in FS decomposition can be written explicitly

$$dL_t = \left((\delta_t^V + \delta_t^D)^\top - \phi_t^\top \sigma_t \right) dW_t + \int_{\mathbf{R}^n} \left((J_t^V + J_t^D)(x) - \phi_t^\top F_t(x) \right) \widetilde{\Pi}(dt, dx) + \sum_{i,j \in \mathcal{K}: j \neq i} \left((\gamma_t^{V,i,j} + \gamma_t^{D,i,j}) - \phi_t^\top \rho_t^{i,j} \right) dM_t^{i,j}$$

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Explicit form of the solution

Proposition

Let $(P^t_s)_{s\geq t}$, $(Z^t_s)_{s\geq t}$ be solution to forward SDE

$$dP^t_s = -P^t_s a^\top_s \left(\sigma_s dW_s + \int_{\mathbf{R}^n} F_s(x)\widetilde{\Pi}(ds,dx) + \sum_{i,j:j\neq i} \rho^{i,j}_s dM^{i,j}_s \right), \quad P^t_t = 1.$$

$$dZ^t_s = -Z^t_s r_s ds, \quad Z^t_t = 1.$$

For every $t \in [\![t,T]\!]$ we have

$$V_t = \mathbb{E}\left(P^t_T Z^t_T h^D_T + \int_t^T P^t_s Z^t_s (g^D_s - a^\top_s A_s(\delta^D, J^D, \gamma^D))\,ds | \mathcal{F}_t \right)$$

Explicit form of the solution

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Let $(P^t_s)_{s\geq t}$, $(Z^t_s)_{s\geq t}$ *be solution to forward SDE*

$$dP^t_s = -P^t_s a^\top_s \left(\sigma_s dW_s + \int_{\mathbf{R}^n} F_s(x)\widetilde{\Pi}(ds, dx) + \sum_{i,j:j\neq i} \rho^{i,j}_s dM^{i,j}_s \right), \quad P^t_t = 1.$$

$$dZ^t_s = -Z^t_s r_s ds, \quad Z^t_t = 1.$$

For every $t \in [\![t, T]\!]$ *we have*

$$V_t = \mathbb{E}\left(P^t_T Z^t_T h^D_T + \int_t^T P^t_s Z^t_s (g^D_s - a^\top_s A_s(\delta^D, J^D, \gamma^D))\, ds | \mathcal{F}_t \right)$$

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Interpretation of the solution

If P^0 is a positive martingale, then

$$V_t = B_t \mathbb{E}_{\mathbb{Q}}\left(\frac{h_T^D}{B_T} + \int_t^T \frac{1}{B_s}(g_s^D - a_s^\top A_s(\delta^D, J^D, \gamma^D))\, ds | \mathcal{F}_t \right)$$

where $\mathbb{Q}$ is an equivalent probability measure

$$\frac{d\mathbb{Q}}{d\mathbb{P}}|_{\mathcal{F}_T} = P_T^0$$

In fact SC condition implies that $\mathbb{Q}$ is then a equivalent martingale measure which is known as minimal martingale measure.

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Lemma

Suppose that P^0 is nonegative. Let $\mathbb{Q}$ be a measure with density given by P^0, then

1. *The process $W^{\mathbb{Q}}$ defined by*

$$W_t^{\mathbb{Q}} = W_t + \int_0^t \theta_u^{\mathbb{Q}} du$$

is $\mathbb{Q}$ Brownian motion, where $\theta_u^{\mathbb{Q}} := (\sigma_u)^\top \alpha_u$.

2. *Integer valued random measure Π has $\mathbb{Q}$-compensator with density given by $\nu_t^{\mathbb{Q}}(dx) = (1 - \alpha_t^\top F_t(x))\nu_t(dx)$ i.e. random measure $\widetilde{\Pi}^{\mathbb{Q}}$ defined by*

$$\widetilde{\Pi}^{\mathbb{Q}}(dx, du) := \Pi(dx, du) - (1 - \alpha_u^\top F_u(x))\nu_u(dx)du,$$

is $\mathbb{Q}$ compensated random measure.

Lemma (cont.)

1. *For every $i, j \in \mathcal{K}$, $i \neq j$ processes*

$$\lambda_t^{\mathbb{Q},i,j} = (1 - \alpha_t^\top \rho_t^{i,j})\lambda_t^{i,j}$$

are intensities of processes $H^{i,j}$ i.e. the processes

$$M_t^{\mathbb{Q},i,j} := H_t^{i,j} - \int_0^t H_{u-}^i \lambda_u^{\mathbb{Q},i,j} du$$

are $\mathbb{Q}$ local martingales.

One may expect that following version of risk neutral valuation formula holds

$$V_t = B_t \mathbb{E}_{\mathbb{Q}}\left(\int_t^T \frac{1}{B_s} dD_s | \mathcal{F}_t \right)$$

Consider special example of dividend process

$$D_t = h_T^D \mathbb{1}_{t \geq T} + \int_0^t g_u du + \int_0^t \sum_{i,j:j \neq i} \gamma_u^{D,i,j} dH_u^{i,j}$$

It is special semimartingale with canonical decomposition given by

$$D_t = h_T \mathbb{1}_{t \geq T} + \int_0^t \left(g_u + \sum_{i,j:j \neq i} \gamma_u^{D,i,j} \lambda_u^{i,j} \right) du + \int_0^t \sum_{i,j:j \neq i} \gamma_u^{D,i,j} dM_u^{i,j}$$

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THANK YOU FOR YOUR ATTENTION !!!